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Investigation of Instantaneous Carrier Phase Ambiguity Resolution with the GPS/GALILEO Combination using the General Ambiguity Search Criterion

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Abstract

The success of high-precision Global Navigation Satellite Systems (GNSS) kinematic positioning depends partly on the ability to resolve the integer phase ambiguities. In this paper, we propose a new algorithm for instantaneous kinematic ambiguity resolution, for present and modernised GPS and for GALILEO. This approach - the GEneral Criterion Cascading Ambiguity Resolution (GECCAR) - selects the integer set of ambiguities using the General Ambiguity Search Criterion (GASC). Simulation runs have shown that single-epoch L1/E1 frequency ambiguity resolution was possible 99% of the considered epochs, when the three frequencies from both systems were used together. This new approach shows an improvement in the selection of the correct set of ambiguities when compared with the selection made by the Integer Least Squares Criterion (ILSC). We conclude that the GECCAR approach is a very promising algorithm for instantaneous ambiguity resolution.

Key words: GNSS, Ambiguity Resolution, Ambiguity Search Criterion

1 Introduction

Global Navigation Satellite Systems (GNSS) allow for high-precision positioning when the carrier phase ambiguities are correctly estimated. Several ambiguity resolution algorithms have been published during the past decades, but the majority of them was developed for application with the two-frequency GPS and is not well suitable for the modernised GPS and the GALILEO. One exception is the LAMBDA (Least-squares AMBiguity Decorrelation Adjustment) method (Teunissen, 1993), widely used for single and dual-frequency GPS processing (Jong *et al*., 1999). Among the ambiguity resolution methods developed for the three frequency systems we can refer the CIR (Cascade Integer Resolution) (Jung, 1999; Jung *et al*., 2000), the TCAR (Three-Carrier Ambiguity Resolution) (Forsell *et al*., 1997), the ITCAR (Integrated Three Carrier Ambiguity Resolution) (Vollath *et al*., 1998) and the Geometry-based Cascading Ambiguity Resolution (Zhang *et al*., 2003) methods. Usually, they were tested in static positioning or in kinematic positioning with short baselines. Xu (2006) presents another approach on ambiguity resolution, based on Voronoi Cells.

In general, the ambiguity resolution methods use the Integer Least Squares Criterion (ILSC) to select the integer ambiguities. This paper presents a new instantaneous ambiguity resolution algorithm, the GEneral Criterion Cascading Ambiguity Resolution (GECCAR), which is based on the General Ambiguity Search Criterion (GASC) and that can be applied to present and modernised GPS and to GALILEO.

The main objective was to create an efficient ambiguity resolution algorithm that enables the GNSS kinematic positioning for medium and long distances between the rover and the reference stations. In order to prove the effectiveness of this algorithm, results obtained using real GPS data and simulated data from the modernised GPS-only system, from the GALILEO-only system and from both systems will be described. In addition, results obtained when the ILSC was used instead of the GASC will also be presented, to make possible an adequate comparison between both criteria solutions.

2 Observation Model and Ambiguity Resolution

The general GNSS observation model, in which code and carrier phase observables are included, can be written in the following system of observation equations:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{v} \qquad (1)$$

with

$$\boldsymbol{A} = \left[\boldsymbol{A}_a \;|\; \boldsymbol{A}_b\right]$$

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{a} \\ - \\ \boldsymbol{b} \end{bmatrix}$$

where $\boldsymbol{y}$ is the observation vector (m×1), $\boldsymbol{x}$ is the complete vector of unknown parameters ((n+t)×1) (comprising the integer double-difference carrier phase ambiguities, $\boldsymbol{a}$ (n×1), and the coordinates and other unknown parameters, $\boldsymbol{b}$ (t×1)), $\boldsymbol{A}$ is the design matrix (m×(n+t)) for the full unknown vector, which can be expressed in terms of the design matrices $\boldsymbol{A_a}$ (m×n) and $\boldsymbol{A_b}$ (m×t), corresponding to vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, and $\boldsymbol{v}$ is the measurement noise vector (m×1).

In general, the estimation criterion for solving the linear system of equations (Eq. (1)) is based on the least squares principle, which satisfies the condition:

$$\min_{\boldsymbol{a},\boldsymbol{b}} \left\| \boldsymbol{y} - \boldsymbol{A}_a\boldsymbol{a} - \boldsymbol{A}_b\boldsymbol{b} \right\|^2_{\boldsymbol{Q}_y} , \qquad (2)$$

where $\|.\|^2_{\boldsymbol{Q}_y} = (.)^T \boldsymbol{Q}_y^{-1}(.)$ and $\boldsymbol{Q_y}$ is the variance-covariance matrix of the double-difference observables. The solution of the condition expressed by Eq. (2) would be an ordinary unconstrained least squares problem if all the parameters were allowed to range through the space of real numbers (Teunissen, 1998); however, there is no standard technique to solve Eq. (2) taking into account that the ambiguities must take integer values. This problem is known as the Integer Least Squares problem (Teunissen, 1993). Its solution is based on the orthogonal decomposition of Eq. (2):

$$\left\| \boldsymbol{y} - \boldsymbol{A}_a\boldsymbol{a} - \boldsymbol{A}_b\boldsymbol{b} \right\|^2_{\boldsymbol{Q}_y} = \left\|\hat{\boldsymbol{v}}\right\|^2_{\boldsymbol{Q}_y} + \left\|\hat{\boldsymbol{a}} - \boldsymbol{a}\right\|^2_{\boldsymbol{Q}_{\hat{a}}} + \left\|\hat{\boldsymbol{b}}(\boldsymbol{a}) - \boldsymbol{b}\right\|^2_{\boldsymbol{Q}_{\hat{b}(a)}} \qquad (3)$$

where $\hat{\boldsymbol{v}}$ is the unconstrained least squares residual vector, $\hat{\boldsymbol{b}}(\boldsymbol{a})$ is the least squares estimate of $\boldsymbol{b}$ conditioned on $\boldsymbol{a}$, and $\boldsymbol{Q}_{\hat{b}(a)}$ is the corresponding variance-covariance matrix. As a consequence, the minimisation of the problem in Eq. (2), taking into account the integer constraint of the ambiguities, can be solved in three steps. In the first step one disregards the integer constrains on the ambiguities and performs a standard least squares adjustment. As a result, one obtains real-value estimates of $\boldsymbol{a}$ and $\boldsymbol{b}$ ($\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$), referred to as the float solution, together with their variance-covariance matrix, $\hat{\boldsymbol{Q}}$:

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \hat{\boldsymbol{a}} \\ \hat{\boldsymbol{b}} \end{bmatrix} = \left(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{y} \qquad (4)$$

$$\hat{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q}_{\hat{a}} & \boldsymbol{Q}_{\hat{a}\hat{b}} \\ \boldsymbol{Q}_{\hat{b}\hat{a}} & \boldsymbol{Q}_{\hat{b}} \end{bmatrix} = \left(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\right)^{-1} \qquad (5)$$

where $\boldsymbol{P} = \boldsymbol{Q}_y^{-1}$. In the second step, using the float estimation of the ambiguities, $\hat{\boldsymbol{a}}$, one estimates the corresponding integer ambiguities, usually by minimising the second term of the right-hand side of Eq. (3) and setting the last term to zero (Teunissen, 2003; Verhagen, 2004). In this way, the criterion used to estimate the integer ambiguities, $\breve{\boldsymbol{a}}$, called as the *Integer Least Squares Criterion* (ILSC), is:

$$\breve{\boldsymbol{a}}_{ILS} = \min_{\boldsymbol{a}\in Z^n} \left\|\hat{\boldsymbol{a}} - \boldsymbol{a}\right\|^2_{\boldsymbol{Q}_{\hat{a}}} . \qquad (6)$$

This second step is called the ambiguity resolution process. Finally, in the third step, one solves for the last term, and the fixed solution of the remaining parameters is:

$$\breve{\boldsymbol{b}} = \hat{\boldsymbol{b}} - \boldsymbol{Q}_{\hat{b}\hat{a}}\boldsymbol{Q}_{\hat{a}}^{-1}\left(\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}\right). \qquad (7)$$

Teunissen (1998) stressed the importance to consider the question whether the estimated set of integer ambiguities, $\breve{\boldsymbol{a}}$, should be accepted or not if it is of poor quality. That conducted to the need of perform a validation of the estimated integer ambiguities. Various validation procedures, based on several statistical tests, have been proposed in the past, in order to validate the estimated integer ambiguities. Some of them give satisfying results and are widely used. However, those statistical tests are based on incorrect assumptions and lack a theoretical basis (Verhagen, 2004). One of the most popular validation procedures is the ratio test in which the statistic is the ratio of the second minimum (*sec min*) quadratic form of the residuals to the minimum (*min*) quadratic form of the residuals. Using this test, if

$$\frac{\left(\hat{\boldsymbol{v}}^T\boldsymbol{P}\hat{\boldsymbol{v}}\right)_{\sec min}}{\left(\hat{\boldsymbol{v}}^T\boldsymbol{P}\hat{\boldsymbol{v}}\right)_{min}} > RT$$

where the *RT* is an empirical critical value (the value 2.0 has been suggested by Euler and Landau (1992) and Wei and Schwarz (1995) and the value 1.44 was proposed by Tiberius *et al*. (1997)), the ambiguity set that generates the minimum quadratic form of the residuals is validated as the correct solution. Otherwise, there is no integer solution for the ambiguities, for the data set used. Although the ratio test is considered a useful validation procedure and is commonly used, with different critical values, it has a problem in the statistical assumption that

the least-squares residuals of the best solution and the second-best solution are independent (Teunissen, 1998).
Xu (2002) proposed a new criterion to select the integer ambiguity set of carrier phase among the search area, based on the fact that the ILSC (Eq. (6)) was not generally optimal. This criterion, called the *General Ambiguity Search Criterion* (GASC), has the form:

$$\min_{\boldsymbol{x}} \left\| \hat{\boldsymbol{x}} - \boldsymbol{x} \right\|^2_{Q_x} , \tag{8}$$

where $\boldsymbol{x} = [\boldsymbol{a}\,\boldsymbol{b}]^T$, $\boldsymbol{a} \in Z^n$, $\boldsymbol{b} \in R^t$, and $\hat{\boldsymbol{x}} = [\hat{\boldsymbol{a}}\,\hat{\boldsymbol{b}}]^T$. This new criterion was developed based on the fact that the third term of the right-hand side of Eq. (3) can not be set to zero (Xu, 2004). Unlike the ILSC, the GASC takes into account the residuals for all the unknowns and not only for the ambiguities, providing a unique and optimal solution under the least squares principle and under the condition of integer ambiguities (Xu, 2007). Therefore, with the given data, following the GASC, there is no need to validate the estimated integer ambiguities, as the estimated set of integer ambiguities is the best one that can be reached.

By diagonalising the normal equations related to the observation equation system (Eq. (1)), which have the ambiguity parameters separated from the other unknowns, a criterion equivalent to Eq. (8), known as *Equivalent General Criterion* (EGC), can be written (Xu, 2004):

$$\min_{\boldsymbol{a}\in Z^n,\, \boldsymbol{b}\in R^t} \left((\hat{\boldsymbol{a}} - \boldsymbol{a})^T \boldsymbol{Q}_{\hat{a}}^{-1} (\boldsymbol{a} - \boldsymbol{a}) + (\hat{\boldsymbol{b}} - \boldsymbol{b})^T \boldsymbol{Q}_{b}^{-1} (\boldsymbol{b} - \boldsymbol{b}) \right). \tag{9}$$

For convenience, by denoting

$$da = (\hat{\boldsymbol{a}} - \boldsymbol{a})^T \boldsymbol{Q}_{\hat{a}}^{-1} (\hat{\boldsymbol{a}} - \boldsymbol{a}) \tag{10}$$

and

$$db = (\hat{\boldsymbol{b}} - \boldsymbol{b})^T \boldsymbol{Q}_{\hat{b}}^{-1} (\hat{\boldsymbol{b}} - \boldsymbol{b}), \tag{11}$$

Eq. (9) may be written as

$$\min_{\boldsymbol{a}\in Z^n,\, \boldsymbol{b}\in R^t} (da + db), \tag{12}$$

where *da* represents an enlarging of the residuals due to the ambiguity change caused by ambiguity fixing, and *db* represents an enlarging of the residuals due to the coordinates change caused by ambiguity fixing.

The EGC provides a way to show the relationship between the ILSC and the GASC: the function to be minimised using the ILSC is just one of the terms of the EGC (Xu, 2002). Thus, minimising *da* is exactly the same as the ILSC. When the results obtained using Eq. (6) are different from those obtained with Eq. (9), the results from the search using Eq. (6) are only sub-optimal, due to the optimality and uniqueness property of Eq. (9) (Xu, 2007).

3 A New Ambiguity Resolution Algorithm

There are two approaches to resolve the ambiguities – a single-epoch (or instantaneous) approach and a multi-epoch approach. In the single-epoch approach the observations from each epoch are processed independently, whereas the multi-epoch approach uses many sequential observations together. When the observations are significantly contaminated by biases, such as multipath, residual atmospheric effects, and satellite errors, it is more difficult to instantaneously resolve the ambiguities correctly. A multi-epoch approach using a recursive implementation is the most reliable way for dealing with the problem. However, the instantaneous ambiguity resolution has several advantages - it is resistant to negative effects of cycle slips or a loss of lock and the changes in the constellation of the tracked satellites do not introduce additional complications to the data processing.

In order to instantaneous resolve the ambiguities for GPS and for GALILEO, a new ambiguity resolution procedure based on the General Criterion was developed and implemented: the GEneral Criterion Cascading Ambiguity Resolution (GECCAR). The GECCAR uses:

1. a cascading procedure;
2. an *a priori* transformation to decorrelate the ambiguities;
3. a search algorithm, where each ambiguity is constrained with the values of previously selected ambiguities;
4. the General Ambiguity Search Criterion or the Equivalent General Criterion for integer ambiguity selection.

The cascading procedure for the three-frequency systems was introduced by Forsell *et al*. (1997), who developed the TCAR (Three-Carrier Ambiguity Resolution) method for the GALILEO carrier phase ambiguity resolution, and by Jung (1999), who suggested the CIR (Cascade Integer Resolution) method for the modernised GPS. Although proposed for different systems, both methods are based on the idea of widelaning to take advantage of the stepwise-improved precision in carrier phase ranges from the longest wavelength to the shortest wavelength. Both methods are geometry-free, instantaneous integer ambiguity resolution methods, using integer rounding. Other approaches have been developed based on this principle, as the ITCAR (Integrated Three Carrier

Ambiguity Resolution) (Vollath *et al*., 1998) and the Geometry-based Cascading Ambiguity Resolution (Zhang *et al*., 2003) methods.

The linear combinations used in the implemented algorithm are based on the set of frequencies established for modernised GPS and for GALILEO (see Table 1) and are presented in Table 2.

The full GECCAR procedure consists of three steps. In the first step, the EWL (Extra Wide Lane) ambiguities are estimated using the most precise pseudorange available and the EWL phase combination as observables. In the second step, with the ranges based on the results obtained in the first step, the WL (Wide Lane), or the ML (Medium Lane), ambiguities are estimated using the most precise pseudorange and the WL, or the ML, phase combination as observables. Finally, in the third step, the L1/E1 ambiguities are estimated – the observables used are the L1/E1, L2/E5b and L5/E5a carrier phases, and the unknown ambiguities are just the L1/E1 ambiguities as the L2/E5b and L5/E5a ambiguities may be written in function of L1/E1, ML, WL and EWL ambiguities and the ML, WL and EWL ambiguities have been estimated in the second and first steps, respectively.

Table 1 Frequencies established for the GALILEO and the modernised GPS.

System	Signal	Frequency (MHz)
GALILEO	E1	1575.42
	E5a	1176.45
	E5b	1207.14
GPS	L1	1575.42
	L2	1227.60
	L5	1176.45

Table 2 Modernised GPS and GALILEO frequency combinations.

Lane	System	Linear Combination	Wave length (m)
EWL	GPS	L2-L5	5.861
	GALILEO	E5b-E5a	9.765
WL	GPS	L1-L2	0.862
	GALILEO	E1-E5b	0.814
ML	GPS	L1-L5	0.751
	GALILEO	E1-E5a	0.751

In each step, we follow the decorrelation process proposed by Teunissen (1993). Before estimating the ambiguities as integers, the float ambiguities, $\hat{\boldsymbol{a}}$, are transformed into an equivalent, but less correlated set of ambiguities, $\hat{\boldsymbol{z}}$, and the corresponding variance-covariance matrix, $\boldsymbol{Q}_{\hat{a}}$, is transformed into the variance-covariance matrix, $\boldsymbol{Q}_{\hat{z}}$, using the Z-transformation:

$$\hat{\boldsymbol{z}} = \boldsymbol{Z}^T \hat{\boldsymbol{a}}; \quad \boldsymbol{Q}_{\hat{z}} = \boldsymbol{Z}^T \boldsymbol{Q}_{\hat{a}} \boldsymbol{Z} .$$

This **Z**-matrix needs to fulfil certain requirements, in order to be admissible, as the integer nature of the ambiguities should be maintained. Therefore, all entries in the **Z**-matrix should be integers and the inverse of the **Z**-matrix should exist and its entries should be integers as well (Teunissen, 1994). The Z-transformation should aim at maximum possible decorrelation of the ambiguities to make the search algorithm more efficient.

After the decorrelation, the search is performed, over the search space region, in order to estimate the correct values of the ambiguity parameter vector. This is done by constraining each ambiguity candidate on the values of the previously selected ambiguities. In order to select the correct ambiguity set, $\breve{\boldsymbol{a}}$, using the Equivalent General Criterion, it should be done, for each candidate $\boldsymbol{a}$

- the computation of *da*;
- the computation of ***b*** and *db*;
- the computation of *da+db*;
- the selection of $\breve{\boldsymbol{a}}$ that minimises *da+db*.

4 Simulation Description

With the purpose of examining the effectiveness of the proposed algorithm for instantaneous ambiguity resolution, for modernised GPS and for GALILEO, a software-based GNSS data simulator was developed in C++. This software simulates the 27 satellite ephemerides for GALILEO, using the parameters given by Zandbergen *et al*. (2004). The GPS satellite positions were based on the final IGS (International GNSS Service) orbits. It was considered that the GST (GALILEO System Time) is synchronised with the GPS time and that the WGS84 is the reference frame for both systems.

The simulator generates pseudorange and carrier phase observables, for the three frequencies anticipated for the modernised GPS and for the GALILEO, by the addition of the errors inherent to double-difference observation (ionospheric and tropospheric delays, multipath error and receiver noise) to the true ranges. The errors are added to the non differenced true ranges between each station and each observed satellite. In the case of the carrier phase observables, an integer ambiguity value was established for every pair station/satellite. All the other sources of error affecting GNSS positioning ambience were not taken into account, as the contribution of those errors is insignificant when the double-difference model is used at the processing software.

The ionospheric errors were generated using Global Ionospheric Maps (GIMs) from the IGS. These maps are produced in a daily basis, and distributed in the IONospheric EXchange (IONEX) format, and represent the Vertical Total Electron Content (VTEC), each 2 hours, at grids with a resolution of 2.5º latitude by 5º longitude. These maps define the global trend of the ionospheric errors in a set of coefficients of a spherical harmonic expansion. This spherical harmonic expansion is then used to define a grid of VTEC values on a two dimensional ionospheric shell above the area of interest. These grid values are interpolated to the pierce point of the observation and the interpolated values are multiplied by an elevation mapping function to give the final ionospheric error (Alves, 2001). As the same IONEX maps were used when the data was processed, a Non Modelled Ionospheric Residual (NMIR) was added to the ionospheric errors, based on the root-mean-square (RMS) maps associated with the GIMs. IGS also provides daily RMS data files with the variance values of VTEC.

Generally, the hydrostatic component of the tropospheric delay in the zenith direction is about 2.3 m at sea-level and represents about 90% of the total tropospheric delay. As found by Mendes (1999), the hydrostatic component of the tropospheric delay can be modeled to sub-millimeter accuracy with the use of prediction models such as Saastamoinen (1973). The highly variable non-hydrostatic tropospheric delay can only be modeled to an accuracy of a few centimeters in the zenith direction. Further error is introduced when the zenith tropospheric delay is mapped to the elevation angle of the satellite with the use of a mapping function. At the simulation, the tropospheric delays were generated using the Saastamoinen (1973) zenith delay models, combined with the Global Mapping Functions (GMFs) (Boehm *et al.*, 2006), to model the elevation dependence of the zenith tropospheric delay. As the same model was used when the data was processed, an error of 5% of the calculated tropospheric delay was inserted into the non differenced simulated observables.

Regarding the receiver noise, 0.01 cycles were added for all the carrier phase observables for both GPS and GALILEO. For the code pseudoranges, the values considered are shown in Table 3. For the multipath error, a value of 0.10 m was established for all the pseudoranges and a value of 0.025 cycles was set for all the carrier phases, for both GPS and GALILEO.

Table 3 Simulated receiver noise.

	Signal	Receiver Noise
GPS Code	L1	0.40 m
	L2	0.30 m
	L5	0.20 m
GALILEO Code	E1	0.25 m
	E5b	0.30 m
	E5a	0.20 m
Phase	All	0.01 cycles

5 Tests and Results

The aim of the tests was the evaluation of the proposed instantaneous ambiguity resolution algorithm - GECCAR – for modernised GPS and for GALILEO, in kinematic positioning, for different ionospheric residual levels and different frequency scenarios. With this purpose, software to process GNSS data was developed in C++. The implemented model for positioning was a geometry-based model with double-difference observables. The existence of common frequencies between GPS and GALILEO was not taken into account when forming the double-difference equations.

The performance of the proposed algorithm was evaluated in terms of the *Percentage of Correct Instantaneous Ambiguity Resolution* (PCIAR) at L1 (for GPS) or E1 (for GALILEO) frequency. This is not a probabilistic value, as the ambiguity resolution success rate generally used. The correct ambiguity values are previously known, so the estimated ambiguity values may be compared with the true ones. The PCIAR is, then, the value that is calculated by dividing the number of epochs where all the ambiguities were correctly fixed by the total number of epochs of the whole data set.

Several experiments have been carried out using different real and simulated observation sessions. The simulated example presented below is a representative subset of all the experiments carried out and its results illustrate the results obtained with all the experiments.

In order to evaluate the proposed algorithm with actual dual-frequency GPS data, results from a test using real data are shown before presenting the results with simulated data. The GPS real data used in this test was collected during a flight in Central Europe, in May 12th 2005, between 12:00 and 14:10, at a rate of 1 Hz. During the flight, the distance between the rover and the reference antennas varied between 200 m and 280 km, approximately.

The GPS real data was processed, using the software described above, for two different scenarios - GPS2, using L1 and L2 GPS real data, and GPS1, using L1-only GPS real data. With a minimum elevation angle of 15º, between 4 and 8 satellites were available. The ionospheric errors were modelated using GIMs. The tropospheric delays were modelated using the Saastamoinen zenith delay models, combined with the GMFs. At the GPS2 scenario, the cascading ambiguity resolution process comprised two steps: it began with the WL ambiguity estimation and then estimated the L1 ambiguity set. When using the GPS1 scenario, the GECCAR degenerated on a scheme with just one step, processing the L1 pseudoranges and carrier phase observables.

With the goal of evaluating the advantage that the GASC has on the proposed ambiguity resolution algorithm, the observations were processed two more times, using the same software but replacing the GASC by the ILSC at the ambiguity resolution function (with RT=1.44 and RT= 2.0), for each scenario. Table 4 lists the PCIAR values obtained for the two scenarios. The second column shows the PCIAR values when the GASC was used and the third and the fourth columns show the PCIAR values using the ILSC, with RT=1.44 and RT=2.0, respectively. The advantage of using the GASC, instead of the ILSC, is clear for the two scenarios - the PCIAR values reached on all the runs when the ILSC was used are smaller than those obtained when the GASC was used.

Table 4 PCIAR values for the real data tests.

SCENARIO	GASC	ILSC (1.44)	ILSC (2.0)
GPS2	97.1	94.3	92.3
GPS1	62.8	57.1	50.3

Figure 1 illustrates the faults in the estimation of the correct ambiguity set, as a function of the distance between the rover and the reference station, for the GPS2 scenario, using either the GASC or the ILSC (with RT=1.44 and RT=2.0). Table 5 shows the analysis of the partial PCIAR values, in function of the distance, for the GPS2 scenario. Each line corresponds to an interval of 10 km distance, along the trajectory, when the partial PCIAR value is different from 100, at that interval, in one of the runs.

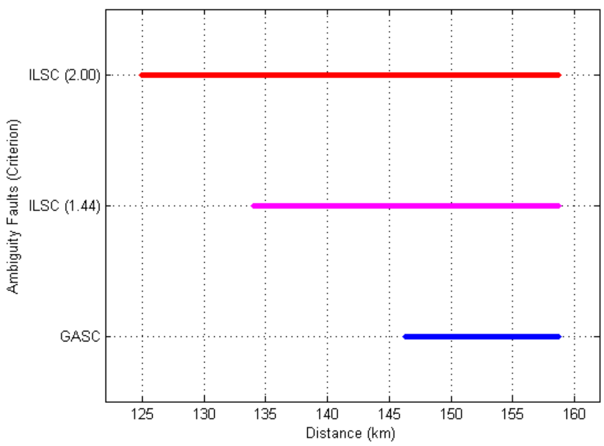

Fig. 1 Faults in ambiguity resolution, using the GPS2 scenario.

Table 5 Partial PCIAR values for the GPS2 scenario.

Criterion / Distance (km)	GASC	ILSC (1.44)	ILSC (2.0)
120 – 130	100.0	100.0	49.4
130 – 140	100.0	39.0	0.0
140 – 150	61.9	0.0	0.0
150 – 160	14.1	14.1	14.1

Note that, due to the way that the atmospheric errors were handled, the ambiguity faults are not function of the distance between the rover and the reference stations; instead, the errors are dependent of the spatial errors of the models. It is clear the advantage of using the cascading scheme for instantaneous ambiguity resolution, even if it comprises only two steps.

For the simulated tests, twelve different scenarios, presented in Table 6, were established. At the GPS3 scenario, that uses simulated L1, L2 and L5 modernised GPS data, the cascading ambiguity resolution process comprises the three steps described above. It begins with the estimation of the EWL ambiguities, then estimates the WL ambiguities and finally estimates the L1 ambiguity set. The GPS2A scenario uses simulated GPS data at L1 and L2 frequencies and the cascading ambiguity resolution process comprises just two steps to estimate the WL ambiguities and the L1 ambiguities. The GPS2B scenario uses simulated L1 and L5 modernised GPS data, and the cascading ambiguity resolution process is identical to the process used with the GPS2A scenario replacing the WL ambiguities by the ML ambiguities. When using the GPS1A scenario, the GECCAR degenerates on a scheme with just one step that processes the L1 pseudoranges and carrier phase observables. The GAL3 scenario contains E1, E5a and E5b simulated data from GALILEO and the cascading process comprises the three steps as the GPS3 scenario. The cascading ambiguity resolution at the GAL2A scenario, which utilises E1 and E5b GALILEO simulated data, is similar to the GPS2A scenario. The GAL2B scenario, where the simulated GALILEO data from E1 and E5a frequencies is used, has the same two steps as the GPS2A scenario. The GAL1A scenario, which uses the E1 frequency data, comprises just one step, as the GPS1A scenario. The GNSS3 scenario uses simulated data from GPS and GALILEO systems at L1, L2, L5, E1, E5a and E5b frequencies. The cascading ambiguity resolution process consists of three steps that estimate firstly the EWL ambiguities for each system, then the WL ambiguities for each system and finally the L1/E1 ambiguities. The L1, L2, E1 and E5b frequency data from GPS and GALILEO are used at the GNSS2A scenario. The two steps of the cascading procedure consist of the WL ambiguity estimation followed by the L1/E1 ambiguity estimation. The GNSS2B scenario uses L1, L5, E1 and E5a frequency data from GPS and GALILEO and the ML ambiguities are estimated before the L1/E1 ambiguity estimation. The GNSS1A scenario comprises just one step to process the L1/E1 data.

The observation session related to the results presented below was simulated based on an aircraft trajectory, at the North Atlantic zone, from (37º 44’ N, 25º 40’ W) to (39º 27’ N, 31º 08’ W), on May 12th 2005, between 12h 00m and 14h 10m. For each GPS and GALILEO satellite above the horizon, pseudorange and phase observables were simulated with a 1 Hz data rate, for the three frequencies. The largest distance between the rover and the reference station was ~512 km. Two levels of NMIR, based on the RMS IONEX maps, were generated: a low level (with 1 RMS) and a medium level (with 2 RMS). The use of RMS maps enables to create an ionospheric error consistent and close to the real situation. The minimum and maximum values concerning the effect of the NMIR on the L1/E1 double difference phase observables, for the different observed satellites, corresponding to 1 RMS, varied between –22.3 cm and 25.4 cm.

Table 6 Simulated data test scenarios.

Scenario	GNSS Type	Phase Frequencies
GPS3	GPS	L1, L2, L5
GPS2A	GPS	L1, L2
GPS2B	GPS	L1, L5
GPS1A	GPS	L1
GAL3	GALILEO	E1, E5a, E5b
GAL2A	GALILEO	E1, E5b
GAL2B	GALILEO	E1, E5a
GAL1A	GALILEO	E1
GNSS3	GPS + GALILEO	L1, L2, L5 + E1, E5a, E5b
GNSS2A	GPS + GALILEO	L1, L2 + E1, E5b
GNSS2B	GPS + GALILEO	L1, L5 + E1, E5a
GNSS1A	GPS + GALILEO	L1 + E1

The simulated data was independently processed twenty four times, using the software described above, for the twelve scenarios and the two levels of NMIR. Using a mask angle of 15º, between 6 and 8 GPS satellites and between 5 and 7 GALILEO satellites were used. With the goal of evaluating the advantage that the GASC has on the proposed ambiguity resolution algorithm, the observations were processed two more times, using the same software but replacing the GASC by the ILSC at the ambiguity resolution function (with RT=1.44 and RT= 2.0), for each scenario and each level of NMIR, using also a 15º mask angle. Table 7 lists the PCIAR values obtained for all the scenarios, using either the GASC or the ILSC, for the two levels of NMIR. It should be pointed out that all the PCIAR values presented are related to the L1/E1 ambiguities. At the steps where the EWL, WL or ML ambiguities were estimated, the corresponding PCIAR values were always 100.

As at the test of real data, the advantage of using the cascading scheme for instantaneous ambiguity resolution is also evident. The results obtained with the GPS1A, the GAL1A and GNSS1A scenarios are always worse than the results obtained with the related scenarios that use two or three frequencies. The merit of using three frequencies is more patent for the medium level of NMIR. Also, the advantage of using the GASC, instead of the ILSC, is clearer for the medium level of NMIR. The PCIAR values reached on all the runs when the ILSC was used are smaller, generally, than those obtained when the GASC was used. For the GNSS scenarios, with a suitable choice of the observed satellites based on its elevation, achievable due to the large number of observed GNSS satellites (between 7 and 12 satellites were available), it was possible to get PCIAR values above 97.7 when two or three frequencies were used together with the GASC, for both levels of NMIR.

Figures 2, 3 and 4 show the values of the distance between the rover and the reference stations related to the epochs where there is a fault in the estimation of the correct ambiguity set, using either the GASC or the ILSC (with RT=1.44 and RT=2.0), obtained when using, respectively, the GPS3, GAL3 and GNSS3 scenarios with the medium NMIR level.

Table 7 PCIAR values for the simulated data tests.

SCENARIO	NMIR LOW			NMIR MEDIUM		
	GASC	ILSC (1.44)	ILSC (2.00)	GASC	ILSC (1.44)	ILSC (2.00)
GPS3	95.8	93.2	89.8	88.5	85.8	82.4
GPS2A	95.4	92.7	89.4	86.8	85.5	82.0
GPS2B	95.3	90.2	85.5	86.2	81.3	73.9
GPS1A	55.2	47.0	28.7	20.2	14.1	10.3
GAL3	100	96.9	93.4	85.3	77.7	66.5
GAL2A	100	88.0	85.1	82.2	66.7	53.7
GAL2B	98.0	85.6	82.8	80.8	55.5	52.0
GAL1A	56.3	37.6	27.7	21.2	16.0	12.2
GNSS3	99.1	99.0	97.0	99.1	96.6	91.2
GNSS2A	99.0	99.0	97.0	98.2	95.7	86.3
GNSS2B	98.7	98.3	96.5	97.7	93.0	80.4
GNSS1A	42.4	33.3	18.7	21.1	15.1	6.2

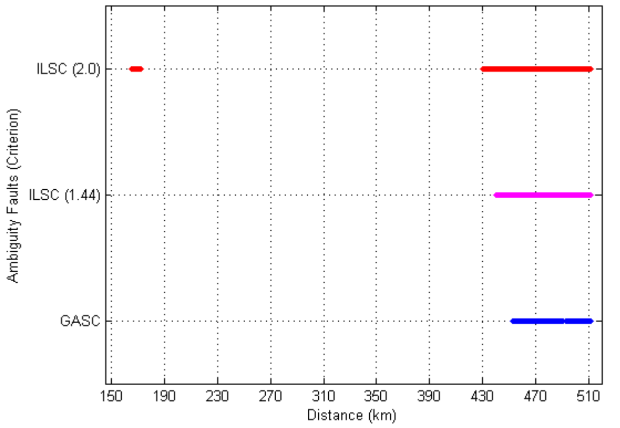

Fig. 2 Faults in ambiguity resolution for the GPS3 scenario, with the medium NMIR level.

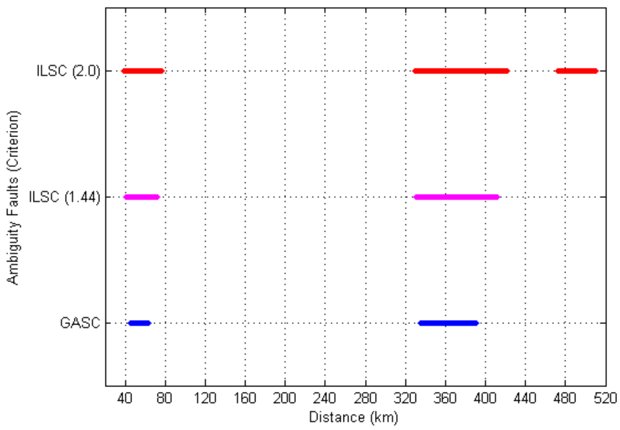

Fig. 3 Faults in ambiguity resolution for the GAL3 scenario, with the medium NMIR level.

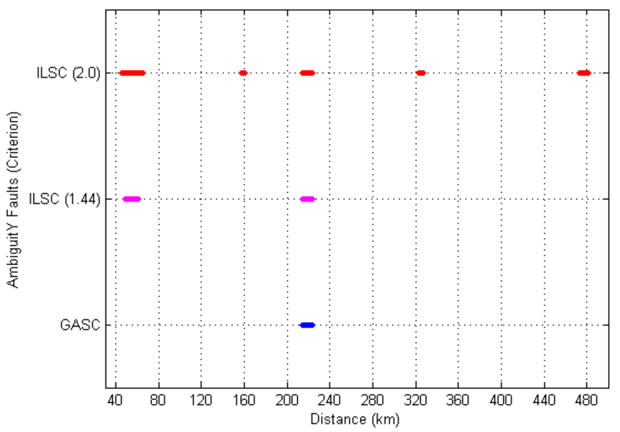

Fig. 4 Faults in ambiguity resolution for the GNSS3 scenario, with the medium NMIR level.

Due to the way that the atmospheric errors were simulated, the ambiguity faults do not increase as the distance between the rover and the reference stations is growing. Tables 8, 9 and 10 list the PCIAR values for each interval of 10 km distance between the rover and the reference stations, where the PCIAR is not 100 for one of the cases (GASC, ILSC with RT=1.44 and ILSC with RT=2), obtained when using the conditions presented in Figures 2, 3 and 4, respectively.

Table 8 Partial PCIAR values for the GPS3 scenario, with the medium NMIR level.

Criterion Distance (km)	GASC	ILSC (1.44)	ILSC (2.0)
160 - 170	100.0	100.0	44.7
170 - 180	100.0	100.0	79.0
430 - 440	100.0	98.1	2.0
440 - 450	100.0	0.0	0.0
450 - 460	22.1	0.0	0.0
460 - 470	0.0	0.0	0.0
470 - 480	0.0	0.0	0.0
480 - 490	0.0	0.0	0.0
490 - 500	12.3	0.0	0.0
500 - 512	0.0	0.0	0.0

Table 9 Partial PCIAR values for the GAL3 scenario, with the medium NMIR level.

Criterion Distance (km)	GASC	ILSC (1.44)	ILSC (2.0)
30 – 40	100.0	98.7	86.8
40 - 50	48.3	0.0	0.0
50 - 60	0.0	0.0	0.0
60 - 70	67.6	0.0	0.0
70 - 80	100.0	70.9	25.2
80 - 90	100.0	57.9	0.0
90 - 100	100.0	100.0	0.0
320 – 330	100.0	100.0	88.2
330 - 340	50.3	5.9	0.0
340 - 350	0.0	0.0	0.0
350 - 360	0.0	0.0	0.0
360 - 370	0.0	0.0	0.0
370 - 380	0.0	0.0	0.0
380 - 390	0.0	0.0	0.0
390 - 400	83.8	0.0	0.0
400 - 410	100.0	0.0	0.0
410 - 420	100.0	83.1	0.0
420 - 430	100.0	100.0	85.0
470 - 480	100.0	100.0	12.3
480 - 490	100.0	100.0	0.0
490 - 500	100.0	100.0	0.0
500 - 512	100.0	100.0	0.0

Table 10 Partial PCIAR values for the GNSS3 scenario, with the medium NMIR level.

Criterion / Distance (km)	GASC	ILSC (1.44)	ILSC (2.0)
40 - 50	100.0	81.5	44.4
50 - 60	100.0	0.0	0.00
60 - 70	100.0	89.4	48.3
150 - 160	100.0	100.0	66.2
160 - 170	100.0	100.0	87.5
210 - 220	71.1	70.4	70.4
220 - 230	82.2	82.2	81.6
320 - 330	100.0	100.0	47.1
470 - 480	100.0	100.0	20.8
480 - 490	100.0	100.0	79.9

In the following, taken from the above processed data, three examples are given to illustrate the behaviour of the two terms of (12). It should be remembered that the sum of these two terms, denoted by *Total*, is the quantity that is used by the GASC to select the correct ambiguity set, and that *da* is the quantity that is used to select the correct set of ambiguities when the ILSC is used.

Tables 11 and 12 are related to the epoch 1010, when data from the GALILEO-only system was used and 6 satellites were observed. Table 11 presents the numbers of the ambiguity set candidates that generated the smallest value for *Total* and the smallest and second-smallest values for *da*. When using the GASC, the selection made was the third candidate as it was the one that generated the smallest *Total*. The choice made by the ILSC was the first one, as it produced the smallest value for *da*. If the ratio test were used, in this case, with a critical value 1.44 or 2.0, the ILSC would reject the found minimum. Table 12 shows the values differences between the true ambiguities and the selected candidate set. The GASC selected the right set of ambiguities. The ILSC choice gave a wrong value for the ambiguities (two of the ambiguities differ one cycle from the true value) but, as this set was not validated, there was no solution, given by the ILSC, for that epoch.

Table 11 Ambiguity candidates, for epoch 1010, using the GAL3 scenario, for medium NMIR.

Candidates	*da*	*db*	*Total*
01	1.720	1.406	3.126
02	2.068	1.694	3.762
03	2.072	0.228	2.300

Table 12 Search result, for epoch 1010, using the GAL3 scenario, for medium NMIR.

Ambiguities	1	2	3	4	5
ILSC selection – 1st candidate	-1	0	1	0	0
GASC solution – 3rd candidate	0	0	0	0	0

Tables 13 and 14 show the results obtained with data from the epoch 1080, also when data from the GALILEO-only system was used and 6 satellites were observed. In this case the ILSC generated the correct set of integer ambiguities but that set was not validated by the ratio test (using RT=1.44 or RT=2.0) and a solution was not given for that epoch. The GASC, with the same candidate vector, because its uniqueness principle, reached the total minimum uniquely and presented the correct solution.

Table 13 Ambiguity candidates, for epoch 1080, using the GAL3 scenario, for medium NMIR.

Candidates	*da*	*db*	*Total*
01	1.300	0.165	1.465
02	1.616	1.330	2.946

Table 14 Search result, for epoch 1080, using the GAL3 scenario, for medium NMIR.

Ambiguities	1	2	3	4	5
ILSC selection – 1st candidate	0	0	0	0	0
GASC solution – 1st candidate	0	0	0	0	0

Tables 15 and 16 describe the selection made by both criteria for the epoch 3440, when data from the GALILEO-only system was used and 6 satellites were observed. Both criteria selected the correct set of integer ambiguities and presented the solution.

Table 15 Ambiguity candidates, for epoch 3440, using the GAL3 scenario, for medium NMIR.

Candidates	*da*	*db*	*Total*
01	0.121	0.068	0.189
02	0.665	0.554	1.219

Table 16 Search result, for epoch 3440, using the GAL3 scenario, for medium NMIR.

Ambiguities	1	2	3	4	5
ILSC solution – 1st candidate	0	0	0	0	0
GASC solution – 1st candidate	0	0	0	0	0

6 Conclusions

The results above show, as similar results from other tests carried out, that the GASC represents a clear improvement in the selection of the correct set of ambiguities. It may be concluded that the GECCAR approach is a very promising algorithm for instantaneous ambiguity resolution. Simulation runs have shown that single-epoch ambiguity resolution was possible 99% of the considered epochs, when the three frequencies from both systems were used together. Benefits from a multi-epochs approach remain to be studied, but a performance gain is expected.

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